Optimal Growth with Public Capital, Public Services and Distortionary Taxation

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The Model

An economy involves a single private good Y which is produced using inputs of primary factors, K and L. Productivity in this activity depends on two types of public goods, a stock good (publically-provided infrastructure, g_s) and a flow good (public services, g_f):

$$Y_t = F(k_t, L_t; g_{st}, g_{ft})$$

I use a semicolon in the function to distinguish private inputs which are remunerated at their marginal product and public inputs which are uncompensated. That is, in equilibrium we have first-order conditions for the wage rate:

$$w_t = p_t(1-\tau)\frac{\partial Y_t}{\partial L_t},$$

and for the rental rate of private capital:

$$r_t = p_t(1-\tau)\frac{\partial Y_t}{\partial k_t}.$$

In these equations p_t is the price of output and τ is the output tax rate.

The public sector is subject to a period-by-period budget constraint, so tax revenue from private production is allocated either to current public services or to expand the provision of public infrastructure:

$$\tau Y_t = i_t^g + g_{ft}$$

Public infrastructure expands with public investment and contracts through geometric depreciation:

$$\dot{g}_{st} = i_t^g - \delta g_{st}$$

Private output which has not been expropriate by government is likewise allocated to current consumption or investment:

$$Y_t(1-\tau) = c_t + i_t$$

Private capital, like public infrastructure expands with private investment and constracts through depreciation:

$$\dot{k}_t = i_t - \delta k_t$$

Finally, following a Ramsey model, investment accomodates a private consumption path which maximizes intertemporal utility subject to a private budget constraint:

$$\max \int_0^\infty e^{-\rho t} \ln c_t \mathrm{d}t$$

s.t.

$$\int_0^\infty p_t c_t = p_0^k k_0 + \int_0^\infty w_t L_t$$

Initial conditions for this model consist of the public capital stock (g_{s0}) and the private capital stock (k_0). Public sector policy involves two decision variables. The first is a scalar, the time-invariant output tax rate, τ. The second is a time-dependent decision rule which determines what fraction of public funds to invest in infrastructure and what faction to spend on current services.